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Code No. : 11221 N/O

**VASAVI COLLEGE OF ENGINEERING (AUTONOMOUS), HYDERABAD**

Accredited by NAAC with A++ Grade

**B.E. I-Semester Main & Backlog Examinations, Jan./Feb.-2024****Calculus & Linear Algebra**

(Common for CSE, AIML &amp; IT)

Time: 3 hours

Max. Marks: 60

Note: Answer all questions from **Part-A** and any **FIVE** from **Part-B****Part-A (10 × 2 = 20 Marks)**

Q. No.	Stem of the question	M	L	CO	PO
1.	Define curvature	2	1	1	1,12
2.	Find the Taylor's series expansion of $\sin x$ about the point $x = \frac{\pi}{2}$	2	2	1	1,12
3.	If $xy^2 \cos xy + x^2y \sin xy = 0$ , find $\frac{dy}{dx}$ .	2	2	2	1,12
4.	If $U = f(x,y)$ , $x = G(s,t)$ , $y = H(s,t)$ , $s = g(r)$ and $t = h(r)$ , then find the total derivative of $U$ with respect to $r$ .	2	3	2	1,12
5.	Is the set of all real numbers $\mathbb{R}$ over the Complex numbers $\mathbb{C}$ , with usual complex multiplication as scalar multiplication, a vector space? Explain	2	4	3	1,2,12
6.	Define the Dimension of a vector space $V(F)$	2	2	3	1,12
7.	Define Null space	2	1	4	1,12
8.	State Rank-Nullity Theorem.	2	1	4	1,12
9.	When a matrix of the linear transformation diagonalizable.?	2	2	5	1,12
10.	If $x = (2, 1 + i, i)$ , $y = (2 - i, 2, 1 + 2i)$ be two elements of the inner product space $C^3(C)$ with respect to the standard inner product, then find $\langle x, y \rangle$ .	2	3	5	1,12
<b>Part-B (5 × 8 = 40 Marks)</b>					
11. a)	Find the radius of curvature at the origin of the curve $y^2 = x^2 \frac{(a+x)}{(a-x)}$	4	2	1	1,12
b)	Find the Taylor's series expansion of $f(x) = \frac{1}{1+x}$ about $x=1$ up to 4 <sup>th</sup> order terms	4	2	1	1,12
12. a)	Find the point on the sphere $x^2 + y^2 + z^2 = 1$ nearest to the point $(2, 1, 1)$ .	4	2	2	1,12
b)	Expand $e^x \sin y$ in powers of $x$ and $y$ as far as terms of third degree.	4	2	2	1,12

Contd... 2

13. a)	Is the set $\{(1, -2, 3), (2, 3, 1), (-1, 3, 2)\}$ a basis for $\mathcal{R}^3$ ?	4	3	3	1,12
b)	If $S = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ is a basis for a finite dimensional vector space $V$ of dimension $n$ , then show that every element of $V$ can be uniquely expressed as a linear combination of the elements of $S$ .	4	3	3	1,12
14. a)	Let $T: \mathcal{R}^4 \rightarrow \mathcal{R}^3$ defined by $T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$ . Find Range( $T$ ), Null space ( $T$ ), rank $T$ and Nullity $T$ .	4	3	4	1,12
b)	Let $T: \mathcal{R}^3 \rightarrow \mathcal{R}^2$ be a linear transformation defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$ . Find the matrix of the linear Transformation $T$ relative to the bases $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $\{(1, 3), (2, 5)\}$ .	4	4	4	1,12
15. a)	From the basis $\{(3, 4, 0), (2, 1, -1), (-2, 1, 3)\}$ , using Gram-Schmidt orthogonalization process construct orthonormal basis in vector space $\mathcal{R}^3(\mathcal{R})$ .	4	3	5	1,12
b)	Find the characteristic values and corresponding Characteristic vectors of the matrix $\begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$	4	2	5	1,12
16. a)	Find the evolute of the curve $x^2 = 4ay$	4	2	1	1,12
b)	If $u = \sin^{-1}(x - y)$ ; $x = 3t$ ; $y = 4t^3$ find total derivative $\frac{du}{dt}$	4	2	2	1,12
17.	Answer any <i>two</i> of the following:				
a)	Is the set of all $2 \times 2$ real matrices over the field of real numbers a Vector space, with respect to addition and multiplication of matrices and scalar multiplication of matrix .?	4	2	3	1,12
b)	Let $T: U \rightarrow V$ be a linear transformation and $S = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ is a basis of $U$ , then show that vectors $T(\alpha_1), T(\alpha_2), T(\alpha_3), \dots, T(\alpha_n)$ generate Range of $T$ .	4	3	4	1,12
c)	Is the matrix $\begin{bmatrix} -6 & 7 & -2 \\ -6 & 7 & 0 \\ 3 & -3 & 5 \end{bmatrix}$ diagonalizable? Explain	4	2	5	1,12

M : Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

i)	Blooms Taxonomy Level - 1	11%
ii)	Blooms Taxonomy Level - 2	52%
iii)	Blooms Taxonomy Level - 3 & 4	37%

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